GLOBAL MATH OLYMPIAD

Past Papers Archive

Compiled from GMO past editions — Seed through Scholar Level

www.mindcarver.com | info@mindcarver.com | +92 313 4506115

MIND CARVER

GLOBAL MATH OLYMPIAD — PAST PAPERS ARCHIVE

Scholar Level

Grades 11–12

Past Paper — For Reference & Practice

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Basic Problems (1 Each - 7 Marks)

1. If $f(x) = x^3$, what is the equation of the function after shifting it 4 units rightwards?

   (a) (A) $x^3 + 4$
   (b) (B) $x^3 - 4$
   (c) (C) $(x + 4)^3$
   (d) (D) $(x - 4)^3$

2. The second derivative of a function at a point provides information about:

   (a) (A) The area under the curve
   (b) (B) The function's slope at that point
   (c) (C) The concavity of the function
   (d) (D) The function's maximum value

3. What is the derivative of $f(x) = \ln(3x)$?

   (a) (A) $\frac{1}{x}$
   (b) (B) $\frac{3}{x}$
   (c) (C) $\frac{1}{3x}$
   (d) (D) $\frac{1}{x+3}$

4. What is the sum of the interior angles of an octagon?

   (a) (A) 1080
   (b) (B) 720
   (c) (C) 540
   (d) (D) 900

5. Which of the following is **not** a measure of dispersion?

   (a) (A) Variance
   (b) (B) Standard deviation
   (c) (C) Range
   (d) (D) Mode

6. The equation of a circle is $(x - 5)^2 + (y + 4)^2 = 49$, what is the centre point of the circle.

   (a) $(0, 0)$
   (b) (± 7)
   (c) $(-5, 4)$
   (d) $(5, -4)$

7. What is the value of $\tan\theta . \cos\theta$

   (a) $\cos\theta$
   (b) $\sin\theta$
   (c) $\sec\theta$
   (d) $\cot\theta$

1 Moderate Problems (2 Each - 48 Marks)

8. The function $h(x) = f(-x)$ represents which transformation of $f(x)$?

   (a) (A) Reflection over the x-axis
   (b) (B) Reflection over the y-axis
   (c) (C) Vertical stretch by a factor of -1
   (d) (D) Horizontal stretch by a factor of 2

9. Which of the following is equivalent to $\log_b(\frac{x}{y})$?

   (a) (A) $\log_b(x) + \log_b(y)$
   (b) (B) $\log_b(x) - \log_b(y)$
   (c) (C) $\frac{\log_b(x)}{\log_b(y)}$
   (d) (D) $\log_b(xy)$

10. The height $h(t)$ of a projectile is modeled by $h(t) = -5t^2 + 30t + 2$. At what time it reaches maximum height?

(a) 3 sec
(b) 4 sec
(c) 5 sec
(d) 6 sec

11. If a company's revenue function is $R(x) = 8x^2 + 4x - 1000$ and a tax of \$5,000 is introduced, what is the new revenue function?

(a) (A) $8x^2 + 4x - 5000$
(b) (B) $8x^2 + 4x + 4000$
(c) (C) $8x^2 + 4x - 4000$
(d) (D) $8x^2 + 4x - 5$

12. The population of a bacteria colony is modeled by $P(t) = 500(1.06)^t$, where *t* is measured in hours. What is the growth rate of the bacteria colony (as a percentage)?

(a) 6%
(b) 500%
(c) 106%
(d) 0.06%

13. Find the derivative of $f(x) = \tan(2x^2 + 1)$ with respect to x.

(a) $2\sec^2(2x^2 + 1)$
(b) $4x\sec^2(2x^2 + 1)$
(c) $\sec^2(4x)$
(d) $8x\sec^2(2x^2 + 1)$

14. Find the derivative of $g(x) = \sqrt{x^3 + 1}$.

(a) $\frac{1}{2\sqrt{x^3+1}}$
(b) $\frac{3x^2}{2\sqrt{x^3+1}}$
(c) $\frac{3x^2}{2\sqrt{x}}$
(d) $\frac{3x^2}{x^3+1}$

15. A boat crosses a river at a speed of 10 m/s and the river flows at 5 m/s. If the boat heads directly across the river, what is the magnitude of the boat's resultant velocity?

(a) 5 m/s
(b) 10 m/s
(c) 11.18 m/s
(d) 15 m/s

16. Find the area of the triangle formed by the vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - 3\hat{j} + 2\hat{k}$.

(a) $\frac{5\sqrt{3}}{2}$
(b) $\frac{4\sqrt{3}}{2}$
(c) $\frac{3\sqrt{3}}{2}$
(d) $\frac{\sqrt{3}}{2}$

17. Simplify the expression $\frac{\sin(2x)}{1+\cos(2x)}$.

(a) $\tan(x)$
(b) $\cot(x)$
(c) $\sin(x)$
(d) $\cos(x)$

18. Solve: $\frac{\tan\theta}{\cot\theta}$.

(a) $\frac{\sin\theta}{\cos\theta}$
(b) $\frac{sin^2\theta}{\cos^2\theta}$
(c) $\sin(x)$
(d) $\cos(x)$

19. A bag contains 5 red balls and 3 blue balls. If two balls are drawn without replacement, what is the probability that both balls are red?

(a) $\frac{5}{8}$
(b) $\frac{5}{14}$
(c) $\frac{25}{64}$
(d) $\frac{10}{56}$

20. A die is rolled twice. What is the probability that the sum of the two rolls is 7?

(a) $\frac{1}{6}$
(b) $\frac{1}{12}$
(c) $\frac{7}{36}$
(d) $\frac{5}{36}$

21. Find the general solution to the equation $\cos(2x) = \frac{1}{2}$.

(a) $x = \pm\frac{\pi}{6} + n\pi$
(b) $x = \pm\frac{\pi}{3} + n\pi$
(c) $x = \pm\frac{\pi}{6} + 2n\pi$
(d) $x = \pm\frac{\pi}{3} + 2n\pi$

22. What is the eccentricity of the hyperbola $\frac{y^2}{9} - \frac{x^2}{16} = 1$?

(a) $\frac{3}{4}$
(b) $\frac{4}{3}$
(c) $\frac{5}{3}$
(d) $\frac{5}{4}$

23. A force of 100 N acts at an angle of 45° to horizontal. What is the vertical component of force?

(a) $50\sqrt{2}$
(b) $100\sqrt{2}$
(c) 50
(d) 100

24. A scientist is modeling the spread of a virus using the exponential function $N(t) = 100e^{0.1t}$ What does the 100 represent in this model?

(a) The initial number of infected people
(b) The infection rate per unit time
(c) The time in hours
(d) The total population limit

25. What is the derivative of $f(x) = e^{3x}\ln(x)$?

(a) $e^{3x}\left(\frac{1}{x} + 3\ln(x)\right)$
(b) $e^{3x}\left(\frac{1}{x} - 3\ln(x)\right)$
(c) $e^{3x}\left(\frac{1}{x} + \ln(x)\right)$
(d) $e^{3x}\left(\frac{1}{x} - \ln(x)\right)$

26. Solve for : $2^{x+1} = 16$

(a) 8
(b) 9
(c) 3
(d) 2

27. The velocity of a moving car is given by function $9t^2 - 6t + 5$. What is the distance covered after 5 seconds.

(a) 375 m
(b) 400 m
(c) 325 m
(d) 125 m

28. Let f be a linear function for which $f(400) - f(200) = 100$. What is $f(x)$?

(a) $2x$
(b) x
(c) $\frac{1}{2}x$
(d) $3x$

29. The function $f(x) = x^4$ has turning points:

(a) 2
(b) 3
(c) 4
(d) 1

30. Integrate the following function: $f(x) = x \sin x$

(a) $-x \cos x - \sin x + c$
(b) $-x \cos x + \sin x + c$
(c) $x \cos x + \sin x + c$
(d) $\sin x$

31. Solve for x in the equation:

$$3^{2x+1} = 81^{x-1}$$

(a) $x = 2$
(b) $x = 3/2$
(c) $x = 4/2$
(d) $x = 5/2$

2 Advanced problems (5 each - 45 Marks)

32. A particle moves along a straight line with position function given by

$$s(t) = t^3 - 6t^2 + 9t + 2,$$

where s is in meters and t is in seconds. At what time t does the particle reach its maximum acceleration?

(a) $t = 0$
(b) $t = 1$
(c) $t = 2$
(d) $t = 3$

33. Solve for x in the equation:

$$\log_3(x + 1) + \log_3(x - 1) = 2$$

(a) $x = 2$
(b) $x = \sqrt{10}$
(c) $x = \sqrt{12}$
(d) $x = 5$

34. In triangle ABC, $AB = 5$, $BC = 7$, and $CA = 8$. What is the area of $\triangle ABC$?

(a) $\sqrt{3}$
(b) $10\sqrt{3}$
(c) $15\sqrt{3}$
(d) 17

35. Let the vectors be defined as follows:

$$\mathbf{a} = (1, 2, 3), \quad \mathbf{b} = (4, 5, 6), \quad \mathbf{c} = (7, 8, 9).$$

Compute the expression:

$$\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$$

(a) 174
(b) 186
(c) 192
(d) 204

36. A dataset of exam scores follows a normal distribution with a mean of 80 and a standard deviation of 10. What is the z-score for a student who scored 50 on the exam?

(a) 0.5
(b) -1
(c) -3
(d) 0.25

37. The equation of a conic section is given by:

$$9x^2 + 4y^2 - 36x + 8y + 4 = 0$$

Which of the following best describes this conic section?

(a) A circle
(b) An ellipse
(c) A parabola
(d) A hyperbola

38. Evaluate the following integral:

$$\int sin^3 x . \cos^2 x dx$$

(a) $-\frac{\sin^3 x}{3} + \frac{\sin^5 x}{5} + c$
(b) $-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$
(c) $-\frac{\cos^4 x}{4} + \frac{\cos^5 x}{5} + c$
(d) $-\frac{\cos^3 x}{3} + c$

39. If $f(x) = x^2 + 2x + 3$. If $f(a) = f(b)$ for some distinct a and b, what is the value of (a+b)?

(a) -2
(b) 0
(c) 2
(d) undefined

40. If $I = \int_0^{\pi} \sin^3 x$, then I =

(a) $\frac{2}{3}$
(b) $\frac{4}{3}$
(c) $\frac{2}{5}$
(d) $\frac{4}{5}$