MIND CARVER

GLOBAL MATH OLYMPIAD

Scholar Level — Grades 11-12

Excellence in Advanced Mathematics

Past Paper — For Reference & Practice

40 Questions · 60 Minutes · 2.5 Marks per Question · 100 Marks Total

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Section 1: Advanced Algebraic Thinking (20 Marks)

Q1. If z = 1 + i, what is the value of z^2 + z-bar (where z-bar is the complex conjugate)?

A. 1 + 3i B. 2 + i C. 3 + i D. 1 + i

Q2. The sum of the roots of the equation x^3 - 6x^2 + 11x - 6 = 0 is:

A. 6 B. 11 C. -6 D. -11

Q3. For what value of k does the equation x^2 - kx + 9 = 0 have exactly one real root?

A. 3 B. 6 C. 9 D. plus or minus 6

Q4. If log base 2 of x plus log base 2 of (x - 2) equals 3, what is the value of x?

A. 2 B. 4 C. 1 D. 3

Q5. The sum of the infinite geometric series 1 + 1/2 + 1/4 + 1/8 + ... is:

A. 2 B. 1.5 C. 1 D. Infinity

Q6. If |2x - 3| < 5, which interval represents all possible values of x?

A. (-1, 4) B. (1, 4) C. [-1, 4] D. (-infinity, 4)

Q7. The polynomial P(x) = x^4 - 5x^2 + 4 has how many real roots?

A. 2 B. 3 C. 4 D. 1

Q8. If a, b, c are in arithmetic progression and a + b + c = 15, what is the value of b?

A. 3 B. 5 C. 15 D. Cannot be determined

Section 2: Calculus Pioneers (20 Marks)

Q9. What is the derivative of f(x) = x^3 * e^x with respect to x?

A. x^2 * e^x * (x + 3) B. 3x^2 * e^x C. x^3 * e^x D. e^x * (x^3 + 3x^2)

Q10. The limit as x approaches 0 of (sin 3x) / x is:

A. 0 B. 1 C. 3 D. Does not exist

Q11. What is the integral of the integral of (2x + 1)^5 dx?

A. (2x+1)^6 / 12 + C B. (2x+1)^6 / 6 + C C. 6(2x+1)^6 + C D. (2x+1)^6 / 2 + C

Q12. If f(x) = ln(x^2 + 1), then f prime of 1 equals:

A. 0 B. 1 C. 1/2 D. 2

Q13. The area bounded by y = x^2, the x-axis, and the lines x = 0 and x = 2 is:

A. 4 B. 8/3 C. 2 D. 8

Q14. For f(x) = x^3 - 3x^2 + 2, the point x = 2 is a:

A. Local maximum B. Local minimum C. Point of inflection D. Neither

Q15. The slope of the tangent to the curve y = x^2 at the point (2, 4) is:

A. 2 B. 4 C. 1 D. 0

Q16. If dy/dx = 2x and y(0) = 3, what is y(2)?

A. 4 B. 7 C. 5 D. 3

Section 3: Geometric Innovators (20 Marks)

Q17. The distance between the points A(3, 4) and B(6, 8) is:

A. 5 B. 7 C. 25 D. sqrt(7)

Q18. What is the equation of a circle with center (2, -3) and radius 4?

A. (x-2)^2 + (y+3)^2 = 16 B. (x+2)^2 + (y-3)^2 = 16 C. (x-2)^2 + (y+3)^2 = 4 D. x^2 + y^2 = 16

Q19. If vectors a = (2, 3) and b = (1, -2), what is a dot b?

A. 8 B. -4 C. 5 D. -1

Q20. The angle between the lines y = x and y = sqrt(3)*x is:

A. 30 degrees B. 45 degrees C. 60 degrees D. 15 degrees

Q21. The volume of a sphere with radius 3 cm is:

A. 36*pi cm^3 B. 27*pi cm^3 C. 12*pi cm^3 D. 108*pi cm^3

Q22. If the midpoint of the line segment joining P(1, 2) and Q(x, y) is (4, 5), what is Q?

A. (7, 8) B. (3, 3) C. (5, 7) D. (2.5, 3.5)

Q23. The eccentricity of the ellipse x^2/25 + y^2/9 = 1 is:

A. 3/5 B. 4/5 C. 16/25 D. 2/5

Q24. A line makes angles 60 degrees, 45 degrees, and theta with the x, y, and z axes respectively. What is cos(theta)?

A. 1/2 B. sqrt(2)/2 C. 1/2 D. 0

Section 4: Trigonometric Masters (20 Marks)

Q25. What is the exact value of sin(75 degrees)?

A. (sqrt(6) + sqrt(2))/4 B. (sqrt(6) - sqrt(2))/4 C. (sqrt(3) + 1)/4 D. 1/2

Q26. If sin(theta) = 3/5 and theta is in the first quadrant, what is tan(theta)?

A. 3/4 B. 4/3 C. 5/3 D. 5/4

Q27. The general solution of sin(x) = 1/2 is:

A. x = n*pi + (-1)^n * pi/6 B. x = 2n*pi +/- pi/6 C. x = n*pi + pi/6 D. x = n*pi/2 + pi/6

Q28. What is the period of the function f(x) = sin(2x) + cos(3x)?

A. pi B. 2*pi C. 3*pi D. 6*pi

Q29. If tan(alpha) + tan(beta) = 5 and tan(alpha) * tan(beta) = 6, what is tan(alpha + beta)?

A. -5 B. 5 C. -1 D. 1

Q30. The principal value of inverse sin(1/2) is:

A. pi/6 B. pi/3 C. pi/4 D. 5*pi/6

Q31. In a triangle ABC, if a = 5, b = 7, and angle C = 60 degrees, what is c^2?

A. 39 B. 74 C. 49 D. 25

Q32. The maximum value of 3*sin(x) + 4*cos(x) is:

A. 3 B. 4 C. 5 D. 7

Section 5: Probability & Statistical Analysis (20 Marks)

Q33. A fair die is rolled twice. What is the probability that the sum is 7?

A. 1/6 B. 1/12 C. 1/36 D. 6/36

Q34. If X follows Normal distribution with mean mu and variance sigma^2, what is P(X = mu)?

A. 0 B. 0.5 C. 1 D. Depends on sigma

Q35. How many ways can 5 people be arranged in a row?

A. 25 B. 120 C. 60 D. 3125

Q36. The mean of the data set {2, 4, 6, 8, 10} is:

A. 5 B. 6 C. 7 D. 8

Q37. If P(A) = 0.4, P(B) = 0.5, and A and B are independent, what is P(A intersection B)?

A. 0.9 B. 0.2 C. 0.1 D. 0.4

Q38. The standard deviation of {1, 1, 1, 1} is:

A. 0 B. 1 C. 2 D. Undefined

Q39. In how many ways can a committee of 3 be chosen from 8 people?

A. 24 B. 56 C. 336 D. 512

Q40. If the correlation coefficient between X and Y is -0.9, this indicates:

A. Strong positive correlation B. Strong negative correlation C. No correlation D. Weak negative correlation